Electro DC

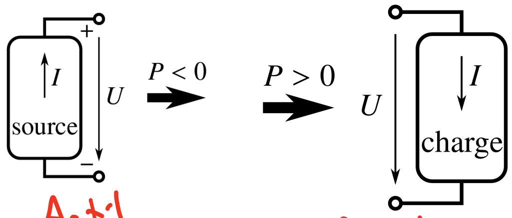
Charge électrique

$e = 1{,}602 \cdot 10^{-19}\,C$ $\quad [C] = [As]$

Courant électrique

Densité courant : $J = \frac{I}{A}$

Dipôles



Actif Passif

Résistance

$R = \frac{U}{I}$ Conductance : $G = \frac{1}{R}$ $[S]$ siemens

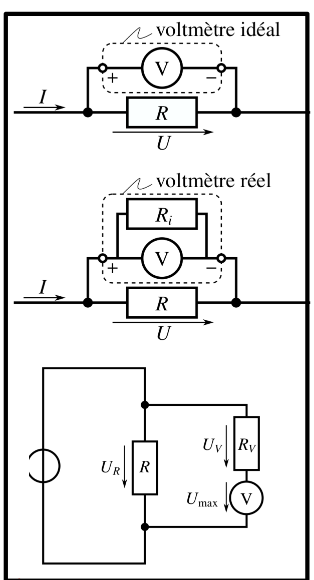
$R = \frac{\rho \cdot \ell}{A}$ $\ell\ [m]$ $A\ [mm^2]$ résistivité : $\rho\left[\frac{\Omega mm^2}{m}\right]$

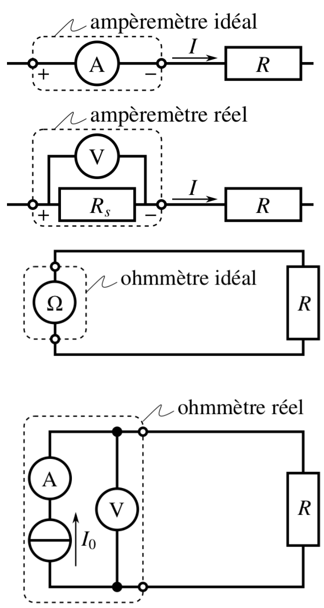
Résistances à coefficient

$\boxed{R(T) = R_{20}\left[1 + \alpha(T-20)\right] \quad \text{PTC}}$

$\frac{R(T_2)}{R(T_1)} = e^{B\left(\frac{1}{T_2} - \frac{1}{T_1}\right)}$ NTC $T \Rightarrow [K]$ kelvin ! B const. $[K]$

Instruments mesure





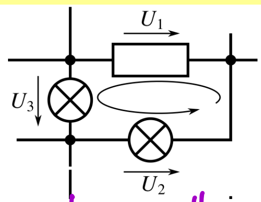
Extension mesure :

$R_V = R_i \frac{U_R - U_{max}}{U_{max}}$

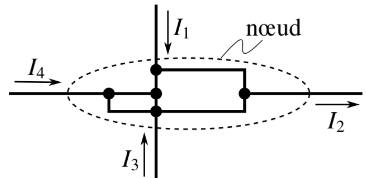
$I_{max} = \frac{U_{max}}{R_i}$

$I_{max} = \frac{U_R - U_{max}}{R_V}$

Lois de Kirchhoff

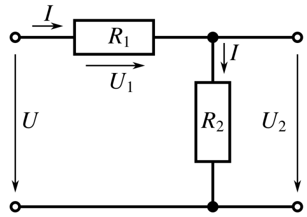


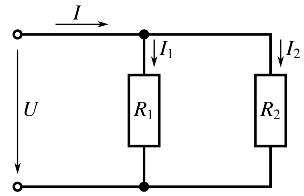
Loi mailles : $\sum_{i=1}^{n} U_i = 0$



Loi noeuds : $\sum_{i=1}^{n} I_i = 0$

Diviseurs de tension et de courant





$$I = \frac{U_2}{R_2} = \frac{U}{R_1 + R_2}$$

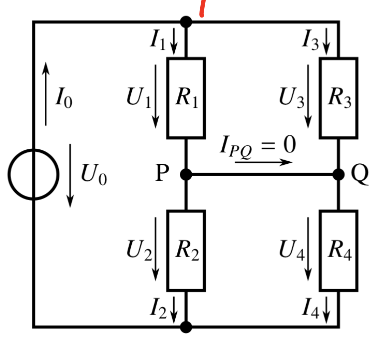
$$U_2 = U\frac{R_2}{R_1 + R_2}$$

$$U = R_2 I_2 = \frac{I R_1 R_2}{R_1 + R_2}$$

$$I_2 = I\frac{R_1}{R_1 + R_2}$$

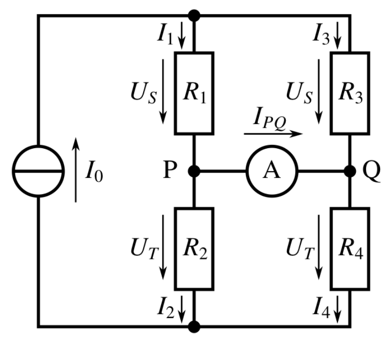
Montage en pont

Pont équilibré



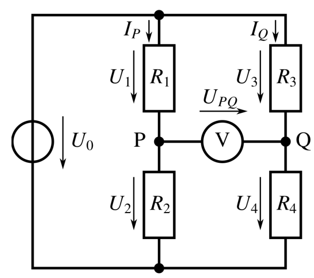
$$\frac{R_1}{R_3} = \frac{R_2}{R_4}$$

Pont non équilibré avec conducteur PQ



$$I_{PQ} = I_1 - I_2 = \frac{U_S}{R_1} - \frac{U_T}{R_2}$$

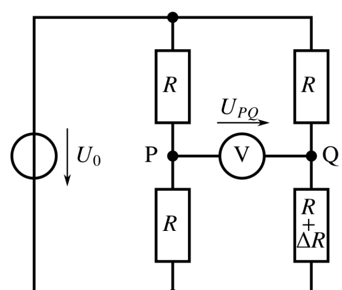
$$I_{PQ} = I_0\left(\frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4}\right)$$



Pont non équilibré sans conducteur PQ

$$U_{PQ} = U_2 - U_4 = R_2 I_P - R_4 I_Q$$

$$U_{PQ} = U_0\left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}\right)$$

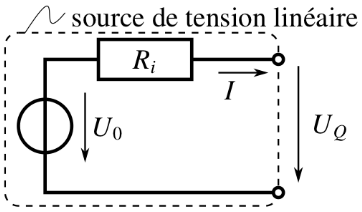


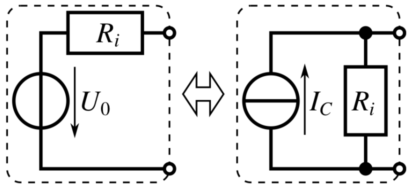
Mesure faible variation de rés

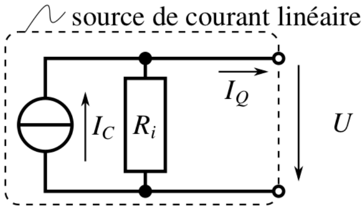
$$R_1 = R_2 = R_3 = R_4 := R$$

$$U_{PQ} = -\frac{U_0}{4}\frac{\Delta R}{R}$$

Sources réelles







Équivalence :

$U_0 = R_i \cdot I_c$

Puissance source de tension

$$P(R_L) = U_0^2 \frac{R_L}{(R_L + R_i)^2}$$

P_{max} quand $R_i = R_L$ $\Longrightarrow$ $P_{\max} = \dfrac{U_0^2}{4R_i}$

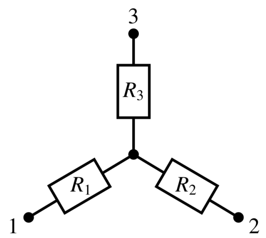
Réseaux de résistances

Transformation triangle – étoile

$$R_1 = \frac{R_{31}R_{12}}{R_{12} + R_{23} + R_{31}}$$

$$R_2 = \frac{R_{12}R_{23}}{R_{12} + R_{23} + R_{31}}$$

$$R_3 = \frac{R_{23}R_{31}}{R_{12} + R_{23} + R_{31}}$$

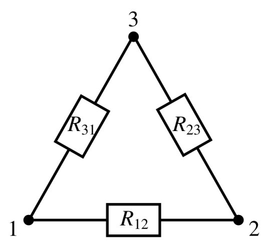


Transformation étoile – triangle

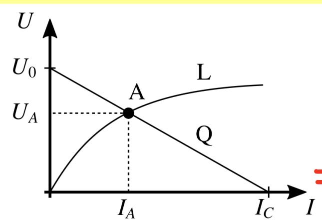
$$R_{12} = R_1 + R_2 + \frac{R_1R_2}{R_3}$$

$$R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1}$$

$$R_{31} = R_3 + R_1 + \frac{R_3R_1}{R_2}$$

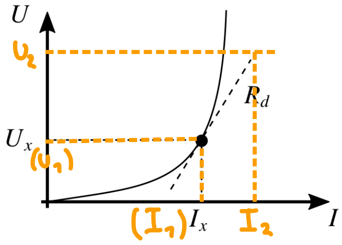


Point de fonctionnement



Superposé la courbe de la source à la courbe de la charge.

$\Longrightarrow$ point de fonctionnement de la charge L



Linéariser une courbe:

$R_s = \dfrac{U_x}{I_x}$ rés. statique à x

$R_d = \dfrac{dU}{dI} = \dfrac{U_2 - U_1}{I_2 - I_1}$ rés. dyn. (pente à x)

$$U(I) = RI_x + R_d(I - I_x)$$

$$U(I) = R_d \cdot I + (R - R_d) \cdot I_x$$

Analyse par superposition

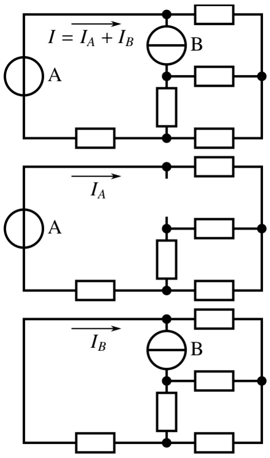
Source de tension

$\Rightarrow$ remplacer par un court-circuit

Source de courant

$\Rightarrow$ interruption

Principe: calculer un courant pour chaque schéma et additionner pour I_{tot}



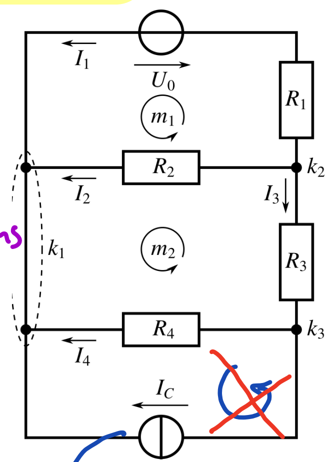
Analyse complète des courants

Il faut:

k-1 équations de noeuds

m équations de mailles

Principe: noter les équations du système et résoudre avec rref la matrice augmentée



$$\begin{pmatrix} 0 \\ I_C \\ U_0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ R_1 & -R_2 & 0 & 0 \\ 0 & +R_2 & -R_3 & -R_4 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix}$$

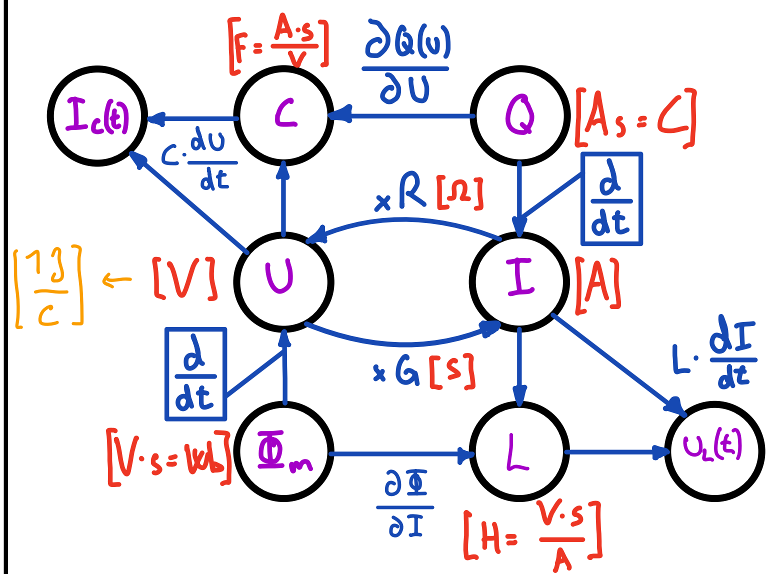
$A_R \cdot I_x = b$

A_R ↓ coefficients des rés.

I_x ↓ vecteur courant à trouver

⚠ la branche contenant une source de courant est factice

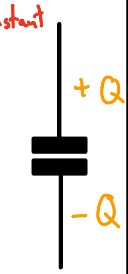
$\Rightarrow$ pas de maille



Condensateur

$[C] = \frac{A \cdot s}{V}$

$C = \underbrace{\frac{\partial Q(U)}{\partial U}}_{\Rightarrow C \text{ pas const.}} = \underbrace{\frac{Q}{U}}_{\text{si linéaire} \Rightarrow C \text{ est constant}}$



$I(t) = \frac{dQ(t)}{dt}$ $\quad I_c(t) = C \cdot \dot{U}_c(t)$

Permittivité

$\varepsilon = \varepsilon_r \cdot \varepsilon_0 \quad \left[\frac{A \cdot s}{V \cdot m}\right]$

$\varepsilon_0 = 8{,}854 \cdot 10^{-12} \frac{A \cdot s}{Vm}$

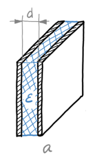
Parallèle / série

$C_{par} = \sum_{i=1}^{N} C_i$

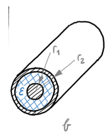
$\frac{1}{C_{ser}} = \sum_{i=1}^{N} \frac{1}{C_i}$

Types de condensateurs

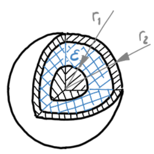
$C'' = \frac{\varepsilon}{d} \quad [F/m^2]$ plan



$C' = \frac{2\pi\varepsilon}{\ln(r_2/r_1)} \quad [F/m]$ Cylindrique



$C = \frac{4\pi\varepsilon}{\frac{1}{r_1} - \frac{1}{r_2}} \quad [F]$ sphérique



Energie et puissance

$W(t) = C \cdot \frac{U^2}{2} = \frac{1}{C} \cdot \frac{Q^2}{2} \quad [J]$

$P(t) = \frac{dW(t)}{dt} = \underbrace{C \cdot \dot{U}(t)}_{I(t)} \cdot U(t) \quad \left[\frac{J}{s} = W\right]$

$Q_C = \frac{U_C^2}{X_C} = X_C \cdot I_C^2 \quad [var]$

Champs magnétique

Champ d'induction magn. (densité ligne champs)

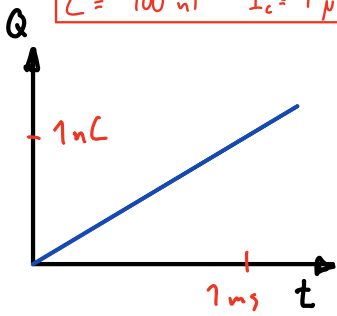
$B = \mu \cdot H \quad [T] = \left[\frac{N}{A^2}\right] \cdot \left[\frac{A}{m}\right] = \left[\frac{Vs}{m^2}\right]$

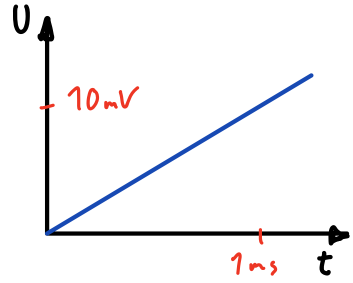
Intensité du champs: $H = \frac{N \cdot I}{\ell} \quad \left[\frac{A}{m}\right] = \left[\frac{[-] \cdot A}{m}\right]$

Flux magnétique

$\Phi = B \cdot S$ surface perp. au flux $[m^2]$

$C = 100\ nF \quad I_c = 1\ \mu A$





$I = \frac{dQ}{dt}$

$\Rightarrow m_Q = \left[\frac{A \cdot s}{s}\right]$

$m_U = \frac{dU}{dt} = \left[\frac{V}{s}\right]$

$m_U = \frac{I}{C} = \left[\frac{A}{\frac{A \cdot s}{V}}\right]$

$\Delta t = \frac{U}{m_U} = \frac{C \cdot U}{I}$

Constante de temps

$\tau = R \cdot C$ $\quad \tau = \frac{|U_{C\infty} - U_{C0}|}{|m_U|}$

résistance équivalente!

Charges

$U_C(t) = U_{C_0} + (U_{C_\infty} - U_{C_0}) \cdot \left\{1 - e^{-\frac{t - t_0}{\tau}}\right\}$

$i_C(t) = I_{C_0} \cdot e^{\frac{-t}{\tau}}$ $\quad t = -R \cdot C \cdot \ln\left(1 - \frac{U_C}{U_{C_0}}\right)$

Décharge

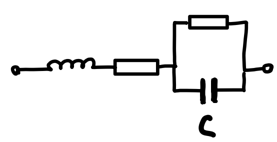
$U_C(t) = U_{C_0} \cdot e^{\frac{-t}{\tau}}$ $\quad t = -R \cdot C \cdot \ln\left(\frac{U_C}{U_{C_0}}\right)$

$i_C(t) = I_{C_0} \cdot e^{\frac{-t}{\tau}}$

Types condensateur :

	Céram.	film plastique	électro-lytique
fréq élevée	+	++	−
Capacité / taille	+	-	++
Variable en température	+	++	−
Capacité	pF ⇝ µF	pF → 100 µF	

Condensateur réel :



Bobine

Perméabilité

$$\mu_0 = 1,256637\cdot 10^{-6}\ \frac{N}{A^2}$$

$$\mu = \mu_0 \cdot \mu_r$$

Couplages

$$L_{ser} = \Sigma_{i=1}^{N} L_i$$

$$\frac{1}{L_{par}} = \Sigma_{i=1}^{N} \frac{1}{L_i}$$

Loi de Faraday

$$U = \frac{d\phi_m}{dt}$$

Flux magnétique emmagasiné

$$\phi_m = L\cdot I \quad [Wb]$$

Induction mutuelle

$$U = \underbrace{\frac{\partial \phi_m (I_2)}{\partial I_2}}_{M(I_2)} \cdot \frac{dI_2(t)}{dt} = M(I_2)\cdot \frac{dI_2(t)}{dt}$$

$$M \propto \mu \cdot N_1 \cdot N_2$$

$$\Rightarrow M = N_1 \cdot N_2 \cdot L_0 \quad [H]$$

$L_0 \triangleq$ inductance pour 1 spire

Auto-induction

$$U = \underbrace{\frac{\partial \phi_m (I)}{\partial I}}_{L(I)} \cdot \frac{dI(t)}{dt} = L(I)\cdot \frac{dI(t)}{dt}$$

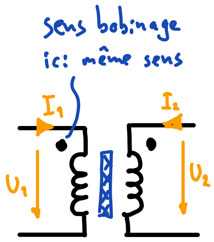
$$L \propto \mu \cdot N^2$$

$$\Rightarrow L_1 = N_1^2 \cdot L_0 \quad [H]$$

Transformateur linéaire

$$U_1(t) = L_1 \cdot \mathring{I}_1(t) + M \mathring{I}_2(t)$$

$$U_2(t) = M \cdot \dot{I}_1(t) + L_2 \dot{I}_2(t)$$



$$-1 \leq k_m = \frac{M}{\sqrt{L_1 \cdot L_2}} \leq +1$$

transfo. sans perte flux

⚠ $M > 0$ si: bobine même sens
sinon $M < 0$

$$U_1(t) = N_1^2 L_0 \cdot \dot{I}_1(t) \pm N_1 N_2 L_0 \cdot \dot{I}_2(t)$$

$$U_2(t) = \pm N_1 N_2 L_0 \cdot \dot{I}_1(t) + N_2^2 L_0 \cdot \mathring{I}_2(t)$$

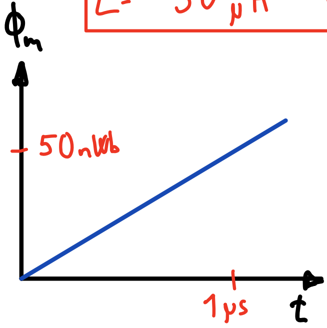
Energie, puissance

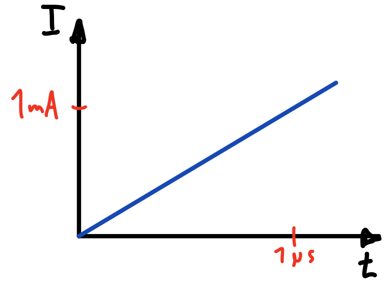
$$P = \frac{dW}{dt} = U \cdot I = L \cdot \dot{I} \cdot I \quad [W]$$

$$W = \frac{1}{2} L \cdot I^2 \quad [J]$$

$$Q_L = \frac{U_L^2}{X_L} = X_L \cdot I_L^2 \quad [var]$$

$$\boxed{L = 50\,\mu H \quad U = 50\,mV}$$





$$U = \frac{d\phi}{dt}$$

$$\Rightarrow m_\phi = U = \left[\frac{V\cdot s}{s}\right]$$

$$m_I = \frac{dI}{dt} = \left[\frac{A}{s}\right]$$

$$m_I = \frac{U}{L} = \frac{V}{\frac{V\cdot s}{A}}$$

$$\Delta t = \frac{I}{m} = \frac{L\cdot I}{U}$$

Constante de temps

$$\tau = \frac{L}{R}$$

résistance équivalente

$$\tau = \frac{|I_{L\infty} - I_{L0}|}{|m_I|}$$

Charges

$$i_L(t) = I_{L0} + (I_{L\infty} - I_{L0})\cdot\left\{1 - e^{-\frac{t-t_0}{\tau}}\right\}$$

$$u_L(t) = U_{L0} \cdot e^{\frac{-t}{\tau}} \qquad t = -R\cdot C\cdot \ln\left(1 - \frac{i_L}{I}\right)$$

U_{L0} → tension au début de la charge sur la bobine

Décharge

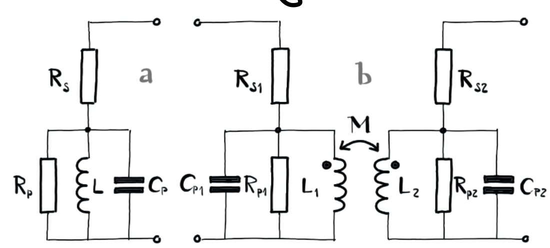
$$i_L(t) = I_{L0} \cdot e^{-\frac{t}{\tau}}$$

$$u_L(t) = U_{L0} \cdot e^{\frac{-t}{\tau}} \qquad t = -R\cdot C\cdot \ln\left(\frac{i_L}{I}\right)$$

Types de bobines:

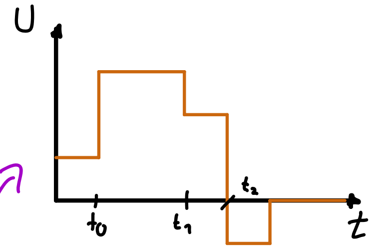
- Bobine à air
- Bobine à noyau

Bobine et transformateur réels:



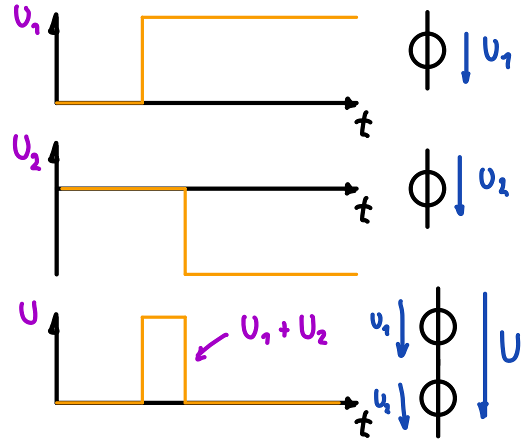
Comportements transitoires 1^{er} ordre

Critères : - une seul bobine ou condo. (linéaires)
- Nombre résistances → illimité
- Nombre sources ayant des sauts → illimité



Principe de superposition

But : transformer 1 source à sauts multiples en multiples sources à 1 seul saut



Evaluation d'un transitoire 1^{er} ordre

1. Evaluer l'état initial pour $t < t_0$
2. Evaluer l'état final pour $t \to \infty$
3. Constante de temps et évolution temporelle

S'il y a un circuit avec composant en série et parallèle => utiliser la loi des noeuds de Kirchhoff pour trouver les courants de condo, bobine

1. Initial $\boxed{t < t_0}$

- Etat stationnaire (tous courants, tensions = constant)
- Courants condo. disparaissent => circuit ouvert $I_C = 0$, $U_C = U_{C_0}$
- Tensions bobines disparaissent => court-circuit $U_L = 0$, $I_L = I_{L_0}$

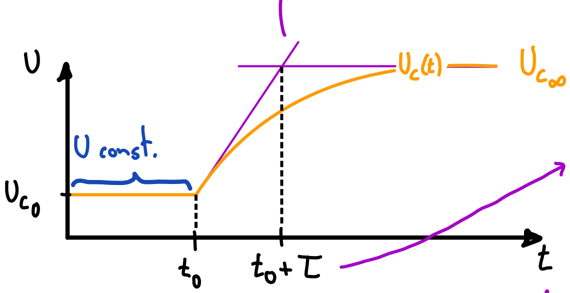
2. Final $\boxed{t \to \infty}$

- Etat stationnaire
- Tension / courant de source après le saut
- Condo → circuit ouvert $I_C(t\to\infty) = 0$, $U_C(t\to\infty) = U_{C_\infty}$
- Bobine → court-circuit $U_L(t\to\infty) = 0$, $I_L(t\to\infty) = I_{L_\infty}$

3. Evolution temporelle (charges)

Condo

$$m_U = \frac{dU_C(t_0)}{dt} = \frac{I_C(t_0^+)}{C}$$



$$\tau = \frac{|U_{C_\infty} - U_{C_0}|}{|m_U|}$$

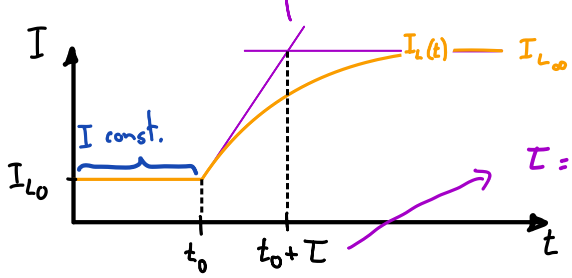
$\tau = R \cdot C$ (R : résistance équivalente)

$$U_C(t) = U_{C_0} + (U_{C_\infty} - U_{C_0}) \cdot \left\{1 - e^{-\frac{t-t_0}{\tau}}\right\}$$

$$i_C(t) = I_{C_0} \cdot e^{\frac{-t}{\tau}} \qquad t = -R\cdot C \cdot \ln\left(1 - \frac{U_C}{U_{C_0}}\right)$$

Bobine

$$m_I = \frac{dI(t_0)}{dt} = \frac{U_L(t_0^+)}{L}$$



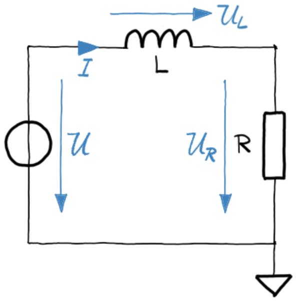
$$\tau = \frac{|I_{L_\infty} - I_{L_0}|}{|m_I|}$$

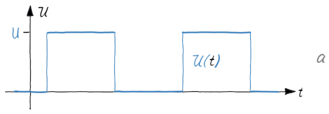
$\tau = \frac{L}{R}$ (R : résistance équivalente)

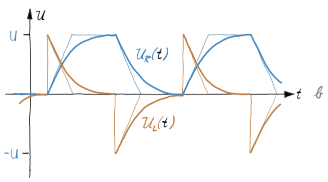
$$i_L(t) = I_{L_0} + (I_{L_\infty} - I_{L_0}) \cdot \left\{1 - e^{-\frac{t-t_0}{\tau}}\right\}$$

$$U_L(t) = U_{L_0} \cdot e^{\frac{-t}{\tau}} \qquad t = -R\cdot C \cdot \ln\left(1 - \frac{i_L}{I}\right)$$

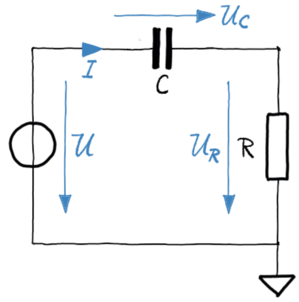
Suppression flanc tension

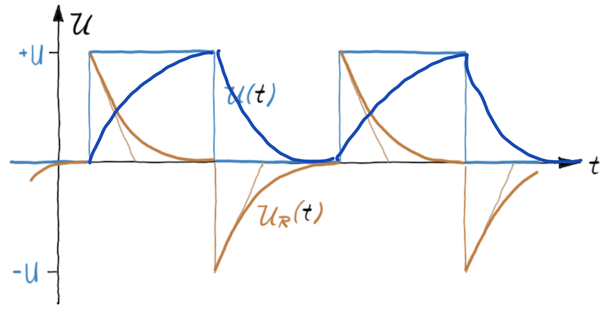


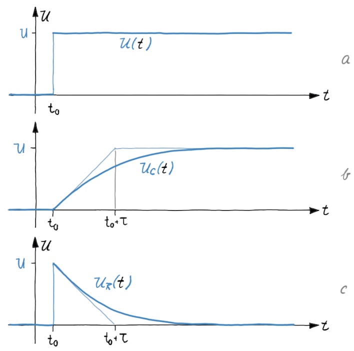


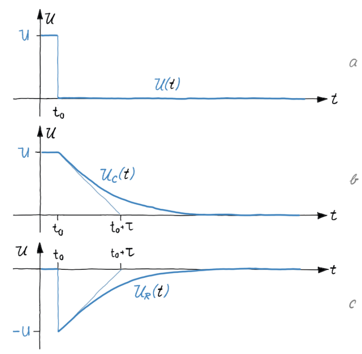


Transmission de flanc de tension









CONDENSATEUR EN COURANT CONTINU

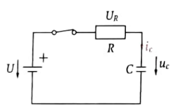
Charge, voir p 21 "champ électrique"

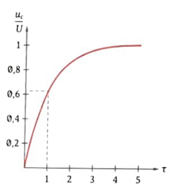
Constante de temps

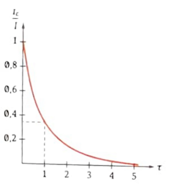
$\tau = R \cdot C$

R	Résistance [Ω]
C	Capacité [F]

Charge







$u_c = U(1 - e^{-t/RC})$

$t = -R \cdot C \cdot \ln\left(1 - \frac{u_C}{U}\right)$

$t = -\tau \cdot \ln\left(1 - \frac{u_C}{U}\right)$

$R = \frac{-t}{C \cdot \ln\left(1 - \frac{u_c}{U}\right)}$

$C = \frac{-t}{R \cdot \ln\left(1 - \frac{u_c}{U}\right)}$

$i_c = I \cdot e^{-t/RC}$

$I = \frac{U}{R}$ à $t = 0$ [s]

$i_c = \frac{U - u_C}{R} = \frac{u_R}{R}$

	i_C	u_C
1 τ	0,37 · I	0,63 · U
2 τ	0,14 · I	0,86 · U
3 τ	0,05 · I	0,95 · U
4 τ	0,02 · I	0,98 · U
5 τ	≅ 0 [A]	≅ U

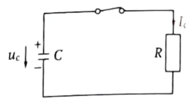
τ	Constante de temps [s]
U_c	Valeur instantanée de la tension aux bornes du condensateur au temps t considéré [V]
i_c	Valeur instantanée du courant dans le condensateur au temps t considéré [A]
t	Temps considéré [s]
e	Base des logarithmes naturels (e = 2,718...)
ln	Logarithme naturel
U	Tension d'alimentation [V]
I	Courant au début de la charge à t = 0 s [A]
U_R	Tension aux bornes de R [V]

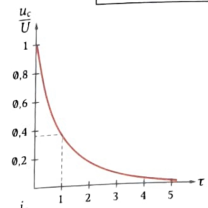
Début de charge
$U_R = U$ et $U_c = 0$ [V]

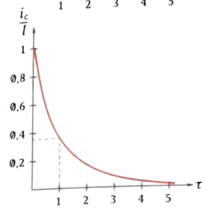
Fin de charge
$U_R = 0$ [V] et $U_c = U$

▸ Condensateur en courant continu

Décharge







$u_c = U \cdot e^{-t/RC}$

$R = \frac{-t}{C \cdot \ln\left(\frac{u_c}{U}\right)}$

$C = \frac{-t}{R \cdot \ln\left(\frac{u_c}{U}\right)}$

$t = -R \cdot C \cdot \ln\left(\frac{u_c}{U}\right)$

$t = -\tau \cdot \ln\left(\frac{u_c}{U}\right)$

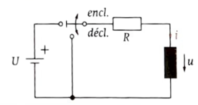
$i_c = I \cdot e^{-t/RC}$

$I = \frac{U}{R}$ et $i_c = \frac{u_c}{R} = \frac{u_R}{R}$

	i_c	u_c
1 τ	0,37 · I	0,37 · U
2 τ	0,14 · I	0,14 · U
3 τ	0,05 · I	0,05 · U
4 τ	0,02 · I	0,02 · U
5 τ	≅ 0 [A]	≅ 0 [V]

u_c	Valeur instantanée de la tension aux bornes du condensateur au temps t considéré [V]
i_c	Valeur instantanée du courant dans le condensateur au temps t considéré [A]
U	Tension aux bornes du condensateur au début de la décharge à t = 0 [s] [V]
t	Temps [s]
e	Base des logarithmes naturels (e = 2,718...)
R	Résistance [Ω]
C	Capacité du condensateur [F]
τ	Constante de temps [s]
I	Courant au début de la décharge à t = 0 [s] [A]
ln	Logarithme naturel

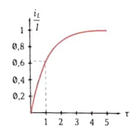
INDUCTANCE EN COURANT CONTINU



$\tau = \frac{L}{R}$

$u_L = U \cdot e^{-t/\tau}$

Enclenchement d'une bobine



$i_L = I(1 - e^{-t/\tau})$

$t = -\tau \cdot \ln\left(1 - \frac{i_L}{I}\right)$

$R = \frac{-L \cdot \ln\left(1 - \frac{i_L}{I}\right)}{t}$

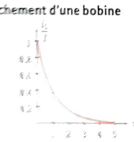
$L = \frac{-R \cdot t}{\ln\left(1 - \frac{i_L}{I}\right)}$

$I = \frac{U}{R}$ et $i_L = \frac{u_R}{R}$

	i_L	u_L
1 τ	0,63 · I	0,37 · U
2 τ	0,86 · I	0,14 · U
3 τ	0,95 · I	0,05 · U
4 τ	0,98 · I	0,02 · U
5 τ	≅ I	≅ 0 [V]

τ	Constante de temps [s]
L	Inductance [H]
R	Résistance série [Ω]
u_L	Valeur instantanée de la tension aux bornes de l'inductance au temps t considéré [V]
U	Tension d'alimentation [V]
t	Temps considéré [s]
i_L	Valeur instantanée du courant dans l'inductance au temps t considéré [A]
I	Courant maximal dans le circuit [A]
ln	Logarithme naturel
e	Base des logarithmes naturels (e = 2,718...)
u_R	Tension instantanée aux bornes de R au temps t considéré [V]

Déclenchement d'une bobine



$i_L = I \cdot e^{-t/\tau}$

$t = -\tau \cdot \ln\frac{i_L}{I}$

$R = \frac{-L \cdot \ln\frac{i_L}{I}}{t}$ $\quad L = \frac{-R \cdot t}{\ln\frac{i_L}{I}}$

Début de l'enclenchement
$u_R = 0$ [V] et $u_L = U$

Fin de l'enclenchement
$u_R \cong U$ et $u_L \cong 0$ [V]

Comportements en AC

Pour $f \to \infty$

Condensateur → court-circuit
bobine → circuit ouvert

Pour $f \to 0$

Condensateur → circuit ouvert
bobine → court-circuit

Courant alternatif

⚠ Calculatrice en radians

Fonction cosinus

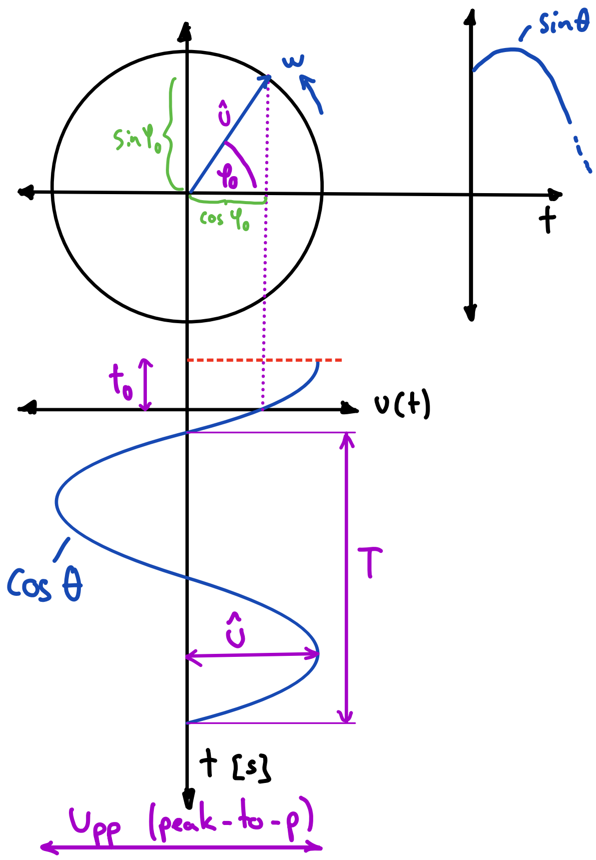
$$\boxed{v(t) = \hat{U} \cdot \cos(\omega t + \varphi_0)}$$

Pulsation : $\omega = 2\pi f$ [rad/s]

fréquence : $f = \frac{1}{T}$ [Hz]

période : T [s]

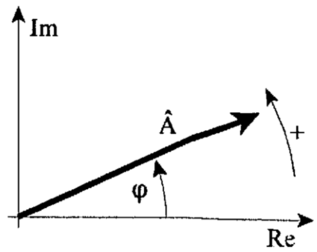
phase : $\varphi_0 = \frac{2\pi}{T} \cdot t_0$ [rad]



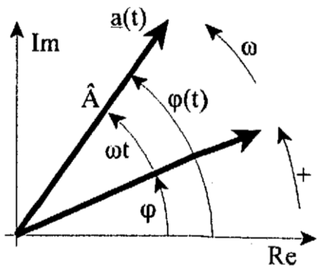
Représentation complexe

Amplitude complexe :

$$\underline{\hat{U}} = \hat{U} \cdot e^{j\varphi_0}$$



Valeur instantanée complexe :
(vecteur complexe)



Polaire :

$$\underline{v(t)} = \hat{U} \cdot e^{j(\omega t + \varphi_0)}$$

$$= \hat{U} \angle (\omega t + \varphi_0)$$

Cartésien :

$$\underline{v(t)} = \underbrace{\hat{U} \cos(\omega t + \varphi_0)}_{Re} + \underbrace{j \sin(\omega t + \varphi_0)}_{Im}$$

$Re = v(t) = \hat{U} \cdot \cos(\omega t + \varphi_0) \Rightarrow$ tension instantanée réelle

Notations complexe : critères

1. Linéarité des composants
2. Invariance temporelles des caractéristiques
3. Sources cosinusoïdals de même fréquence
4. Régime cosinusoïdal établi
   ↳ disparition des transitoires

Impédance

$$Z = \frac{\underline{\hat{U}}(t)}{\underline{\hat{I}}(t)} = \frac{\underline{\hat{U}} \cdot e^{j\omega t} \cdot e^{j\phi_u}}{\underline{\hat{I}} \cdot e^{j\omega t} \cdot e^{j\phi_i}} = \frac{\underline{\hat{U}}}{\underline{\hat{I}}} \cdot e^{j(\phi_u - \phi_i)}$$

$\arg(Z)$ = déphasage

$|Z|$ = amplitude

$i(t) = \underline{\hat{I}} \cdot \cos(\omega t + \phi_i)$ $\quad v(t) = \underline{\hat{U}} \cdot \cos(\omega t + \phi_u)$

$$\boxed{\underline{Z}_C = \frac{-j}{\omega \cdot C} = \frac{1}{j \omega \cdot C} \qquad \underline{Z}_L = \omega \cdot L \cdot j \\ \phi_i = \phi_u - \frac{\pi}{2} \qquad \phi_u = \phi_i - \frac{\pi}{2}}$$

$\omega = 2 \cdot \pi \cdot f$

$\cos(\omega t) = \sin(\omega t + \frac{\pi}{2})$

$-\frac{j}{\omega C} \cdot \frac{j}{j} = \frac{1}{j \omega C}$

Série :

$$\underline{Z} = \sum_{i=1}^{M} \underline{Z}_i$$

Parallèle :

$$\frac{1}{\underline{Z}} = \sum_{i=1}^{n} \frac{1}{\underline{Z}_i}$$

Admittance

$$\underline{Y} = \frac{1}{\underline{Z}}$$

Valeur efficace → valeur crête $\left\{ \begin{array}{l} \hat{U} = \sqrt{2} \cdot U \\ \underline{\hat{U}} = \sqrt{2} \cdot \underline{U} \end{array} \right.$

Kirchhoff complexe

$$\sum_{k=1}^{M} \underline{\hat{I}}_k = 0 \qquad \sum_{k=1}^{N} \underline{\hat{U}}_k = 0$$

Principe de superposition

Reste valable en complexe !

Les grandeurs continues sont remplacées par des amplitudes ou valeurs complexes

- Itération pour chaque source puis addition des courants et tension

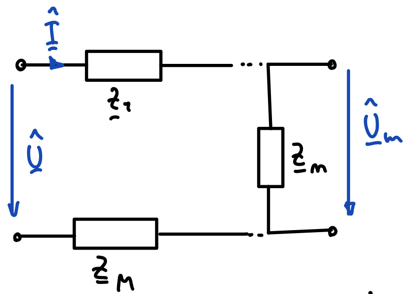
 => annuler par court-circuit

 => annuler par interruption

Transformation de circuits

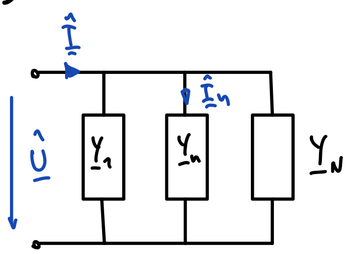
Equivalence des sources $\qquad \underline{\hat{U}}_0 = \underline{\hat{I}}_c \cdot \underline{Z}_i$

Diviseur de tension



$$\frac{\underline{\hat{U}}_m}{\underline{\hat{U}}} = \frac{\underline{Z}_m}{\sum_{i=1}^{M} \underline{Z}_i}$$

Diviseur courant



$$\frac{\underline{\hat{I}}_n}{\underline{\hat{I}}} = \frac{\underline{Y}_n}{\sum_{i=1}^{N} \underline{Y}_i}$$

Dérivation / intégration complexe

$$\frac{d}{dt} \longrightarrow \cdot j2\pi f \qquad \text{multiplier par } j2\pi f$$

$$\int dt \longrightarrow \frac{1}{j2\pi f}$$

Fonctions de transfert

- Rapport des amplitudes complexes de deux variables d'un circuit
- L'amplitude $\underline{A}_v$ est exprimée par son niveau en dB : $L_v = 20 \log_{10} |\underline{A}_v|$
- La phase est exprimée en radians.

	$\underline{\hat{U}}_i$	$\underline{\hat{I}}_i$
$\underline{\hat{U}}_o$	1	3
$\underline{\hat{I}}_o$	4	2

1. $\underline{A}_v(f) = \dfrac{\underline{\hat{U}}_o}{\underline{\hat{U}}_i}$ → gain tension
2. $\underline{A}_I(f) = \dfrac{\underline{\hat{I}}_o}{\underline{\hat{I}}_i}$ → gain courant
3. $\underline{Z}(f) = \dfrac{\underline{\hat{U}}_o}{\underline{\hat{I}}_i}$ → transimpédance
4. $\underline{Y}(f) = \dfrac{\underline{\hat{I}}_o}{\underline{\hat{U}}_i}$ → transadmittance

→ souvent $\underline{H}(f) = \ldots$

Factorisation

Toute fonction de transfert est une fonction rationnelle de l'argument $j2\pi f$ (Puissances 0, 1, 2..) à coefficients réels (produit ou rapport de R, C, L)

$$\underline{A}_v(f) = A_0 \, \frac{1 + a_1 j2\pi f + a_2 (j2\pi f)^2 + \ldots}{1 + b_1 j2\pi f + b_2 (j2\pi f)^2 + \ldots}$$

Il existe trois types de facteurs qui permettent la représentation graphique

1. jf/f_1 $\qquad f_1, f_2$: Fréquences caractéristiques dépendant de R, L, C du circuit
2. $1 + jf/f_2$
3. $1 + jf/(f_3 \cdot Q) + (jf/f_3)^2$

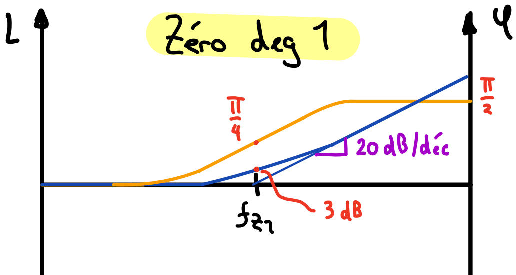
Q: Facteur de qualité (Sans dimension, dépend de R, L, C)

Représentation des fonctions de transfert (ex : circuit passe-bande, passe-bas..)

=> Diagramme de Bode

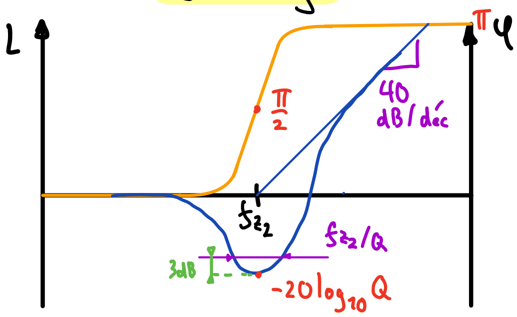
- Axe de fréquence logarithmique
- Amplitude $\underline{A}_v$ exprimée par son niveau L_v [dB] $L_v = 20 \log_{10} |\underline{A}_v|$ — module complexe
- Phase en radians

Zéro deg 1



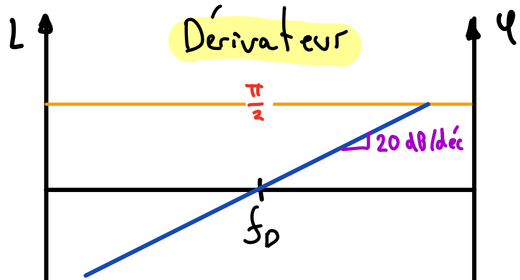
$$1 + jf/f_{z_1}$$

Zéro deg 2



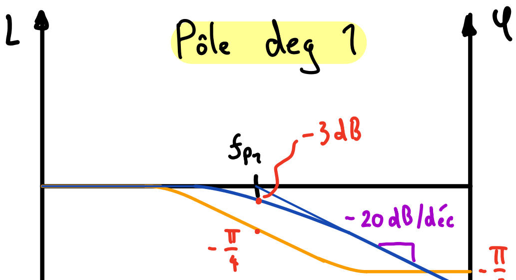
$$1 + jf/(f_{z_2} \cdot Q) + (jf/f_{z_2})^2$$

Dérivateur



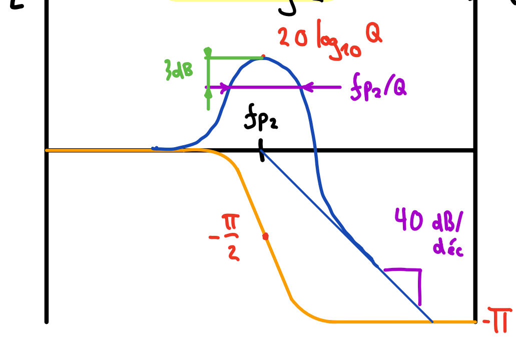
$$jf/f_D$$

Pôle deg 1



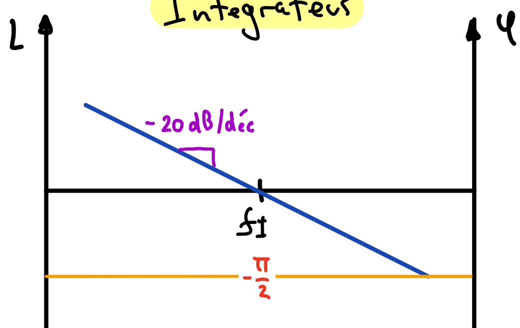
$$1/(1 + jf/f_{p_1})$$

Pôle deg 2



$$\frac{1}{[1 + jf/(f_{p_2} \cdot Q) + (jf/f_{p_2})^2]}$$

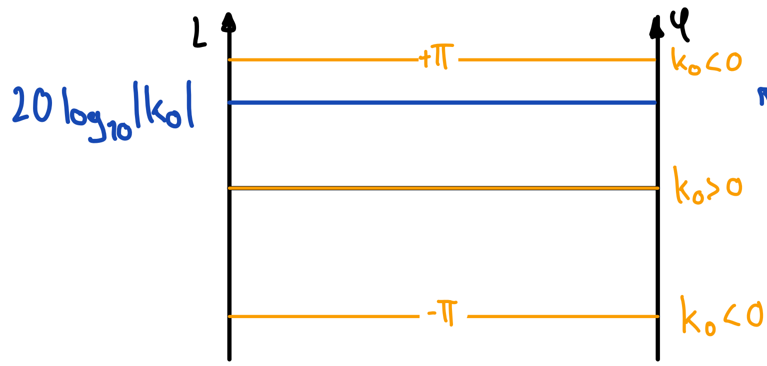
Intégrateur



$$\frac{1}{(jf/f_I)}$$

Facteur Q élevé dit
=> fort pique d'amplitude
et faible bande passante,
pente de la phase est forte

Facteur constant k_0 (ex: $\underline{H}(f) = k_0 \cdot \frac{jf/f_0}{1 + \dots}$)



Bande passante à 3 dB d'un pôle/zéro de degré 2

$$\Delta f_{3dB} = \frac{f_{p_2}}{Q} \quad \text{ou} \quad \Delta f_{3dB} = \frac{f_{z_2}}{Q}$$

Représentation en niveau et phase

Le niveau et la phase d'une fonction de transfert sont la somme des niveaux et phases de ses facteurs.

=> Addition des comportements asymptotiques..

Module et arg de la fonction de transfert

$|\underline{H}(f)|$ indique le gain en fonction de la fréq.

$\varphi = \arg(\underline{H}(f))$ indique le déphasage entre entrée et sortie

Evaluer la fonction de transfert

Evaluer pour $f \to \infty$ ou $f \to 0$

Evaluer directement dans la fonction de transfert

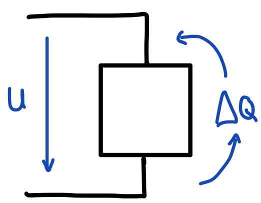
$$G(f\to\infty) = \frac{R_1}{R_1+R_2}\cdot\frac{\left(\cancel{1}+\underbrace{\cancel{j2\pi f}\,\frac{L}{R_1}}_{\to\infty}\right)}{\left(\cancel{1}+\underbrace{\cancel{j2\pi f}\,\frac{L}{R_1+R_2}}_{\to\infty}\right)}$$

$$= \frac{R_1}{R_1+R_2}\cdot\frac{R_1+R_2}{R_1} = \underline{1}$$

$$20\log|G(f\to\infty)| = 0\,dB$$

Puissance et énergie

Cas général :



L'énergie d'un bipôle augmente si sa charge ou sa tension augmente.

⚠ La charge dépend aussi de la tension et vice-versa.

$U(Q)$, $Q(U)$

Changement de tension:

$U = U(Q)$

$$W = \int_0^Q U(\tilde Q)\,d\tilde Q$$

$\tilde Q$ variable d'intégration

Changement de charge:

$Q = U(Q)$

$$W = \int_0^U Q(\tilde u)\,d\tilde u$$

$\tilde u$ variable d'intégration

Si $W > 0 \to$ le bipôle gagne en énergie

Puissance cas général :

$$P = \frac{dW(t)}{dt} \Leftrightarrow W(t) = \int_{-\infty}^{t} P(\tilde t)\,d\tilde t$$

Puissance moyenne dans un interval temporelle

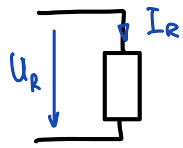
$$\bar P(t_1,t_2) = \frac{1}{t_2-t_1}\int_{t_1}^{t_2} P(t)\,dt$$

$\bar P$ avec signal périodique

$$\bar P = \frac{1}{T}\int_{t_0}^{t_0+T} P(t)\,dt$$

Energie et puissance → signal cosinusoïdal

Résistance



$$P(t) = \frac{\hat U_R^2}{2R}\cdot\cos^2(2\pi\cdot f\cdot t+\varphi_{U_R})$$

Puissance moyenne : $\bar P_R = \frac{\hat U_R^2}{2R} = \frac{U_R^2}{R}$ $\quad U = \frac{\hat U}{\sqrt2}$

Impédance Z

$\underline Z = Z\cdot e^{j\varphi}$

$u(t) = \hat U\cdot\cos(2\pi f t)$

$I(t) = \left(\frac{\hat U}{Z}\right)\cdot\cos(2\pi f t-\varphi)$

$$P(t) = \frac{\hat U^2}{2Z}\left[\cos\varphi + \cos(2\pi\cdot 2\cdot f\cdot t-\varphi)\right]$$

$$\bar P = \frac{\hat U^2}{2Z}\cdot\cos\varphi = \frac12 Z\hat I^2\cos\varphi$$

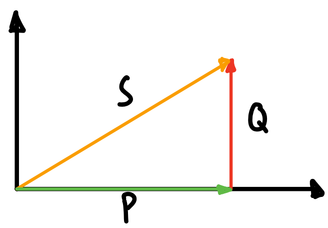
$$\bar P = \frac{|\underline U|^2}{Z}\cos\varphi = Z\cdot|\underline I|^2\cos\varphi$$

↳ module complexe

Puissance active $P = |\underline U|\cdot|\underline I|\cdot\cos\varphi$

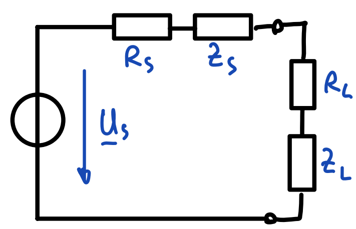
Puissance réactive $Q = |\underline U|\cdot|\underline I|\cdot\sin\varphi$

Puissance apparente $S = |\underline U|\cdot|\underline I|$



Adaptation de puissance

Puissance max disponible :



$$P_{av} = \frac14\cdot\frac{|\underline U_s|^2}{R_s}$$

avec $R_L = R_s$

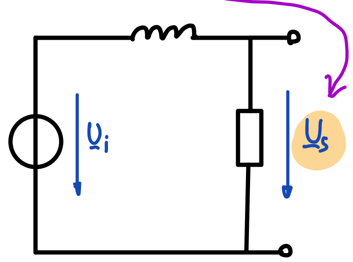
$X_L = -X_s$

⇓

et $\underline Z_L = \underline Z_s^*$

conjugué complexe

⚠ U_s : tension de source à vide



$$\underline Z_s = \frac{(2\pi f)^2 RL^2}{R^2+(2\pi f L)^2} + j\,\frac{2\pi f R^2 L}{R^2+(2\pi f L)^2}$$

$$P_{Av} = \frac14\cdot\frac{R^2+(2\pi f L)^2}{(2\pi f)^2 R L^2}\cdot\frac{\hat U_i^2}{2}$$

Méthode nodale modifiée

1. Choix d'un noeud de référence
2. Définir les potentiels des autres noeuds par rapport au potentiel de référence
3. Pour chaque source de tension idéale, définir son courant comme inconnue additionnelle
4. Poser les équations de Kirchhoff des noeuds en terme des potentiels des noeuds, des courants des sources de courant (Paramètres connus) et des courants des sources de tension (inconnus)
5. Exprimer les tension des sources idéales de tension en terme des potentiels de noeuds

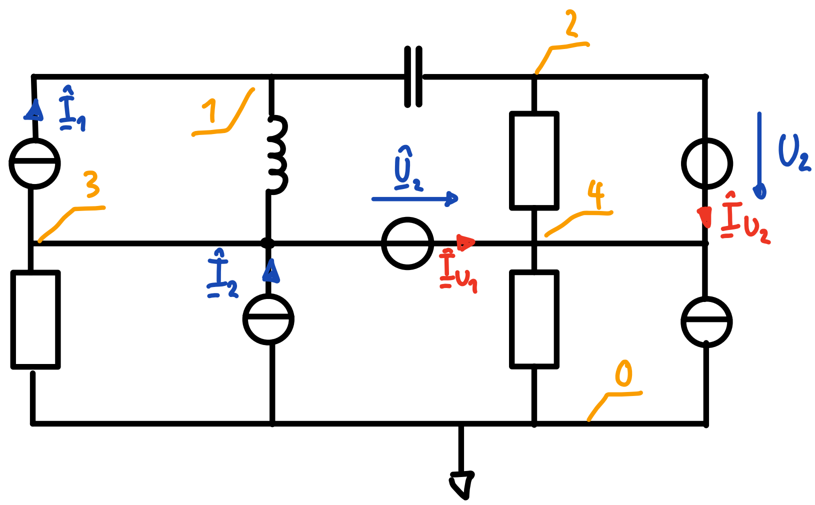
Fonctionne aussi avec des sources cosinusoidales

Circuit à N noeuds —>

N-1 équations nodales en N-1 potentiels et M courants de sources de tension

M-1 équations supplémentaires en les potentiels de noeud exprimant les tension de source.

Le noeud de référence est arbitraire, on prendra celui qui a le plus de connexions !



Consider the circuit shown at figure 1 composed of the current-source providing the time-dependent current $I_S(t)$, of the two resistors with resistances R_1 = 22 kΩ, R_2 = 10 kΩ and of a coil-inductor with inductance L = 22 mH. The source-current's initial-value is $I_{S.0}$ = 15 µA and it jumps at t_0 abruptly to its final value $I_{S.\infty}$ = 55 µA.

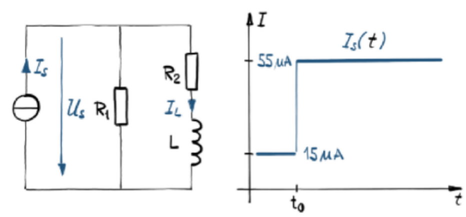


Figure 1: Schematic of Circuit of Exercise Nº 1 on transient Behaviour

1. Determine the initial state $I_{L.0}$ of the inductor-current $I_L(t)$ for $t < t_0$.
2. Determine the final state $I_{L.\infty}$ of the inductor-current $I_L(t)$ for $t \to \infty$.
3. Determine the inductor-voltage $U_L(t_0^+)$ immediately after the abrupt change of source-current and the inductor-current's slope m_I at the switching-instant t_0.
4. Determine the time-constant τ of the circuit's settling-behaviour and represent the inductor-current $I_L(t)$ graphically as function of time t on the empty chart herebelow (figure 2).
5. Represent the source-voltage $U_S(t)$ as function of time on the empty chart herebelow (figure 3).

1. $I_{L.0} = R_1 \cdot I_{S.0}/(R_1 + R_2) = 10.3\ \mu A$
2. $I_{L.\infty} = R_1 \cdot I_{S.\infty}/(R_1 + R_2) = 37.8\ \mu A$
3. $I_L(t)$ is continuous over time, especially at t_0 $\Rightarrow$ $I_S(t_0^+) = I_{R1}(t_0^+) + I_{R1}(t_0^+)$ $\Rightarrow$ $I_{S.\infty} = U_S(t_0^+)/R_1 + I_{L.0}$ $\Leftrightarrow$ $U_S(t_0^+) = R_1(I_{S.\infty} - I_{L.0})$ On the other hand one has $U_S(t_0^+) = U_{R2}(t_0^+) + U_L(t_0^+)$ $\Rightarrow$ $R_1 \cdot (I_{S.\infty} - I_{L.0}) = R_2 I_{L.0} + U_L(t_0^+)$ $\Leftrightarrow$ $U_L(t_0^+) = R_1 \cdot (I_{S.\infty} - I_{L.0}) - R_2 I_{L.0} = 880\ mV$ Finally $m_I = U_L(t_0^+)/L = 40.0\ \mu A/\mu s$
4. $\tau = |I_{L.\infty} - I_{L.0}|/m_I = 688\ ns$ Alternative evaluation using equivalent resistance seen by coil-inductor: $\tau = L/(R_1 + R_2) = 688\ ns$ See below for the graphical representation of the inductor-current (figure 4).
5. $I_S = I_{R1} + I_L$ $\Rightarrow$ $U_S = R_1 \cdot (I_{S.\infty} - I_L)$ See below for the graphical representation of the source-voltage (figure 5).

Exercise Nº 2

Given the circuit shown at figure 2.a and the time-dependent source-voltage U_s shown at figure 2.b

a) evaluate the initial state $I_{L.0}$ of the inductor-current I_L and $U_{o.0}$ of the output-voltage U_o;

b) evaluate the final state $I_{L.\infty}$ of the inductor-current I_L and $U_{o.\infty}$ of the output-voltage U_o;

c) evaluate the time-dependency of the output-voltage U_o and its time-constant τ.

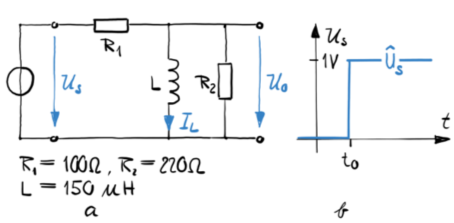
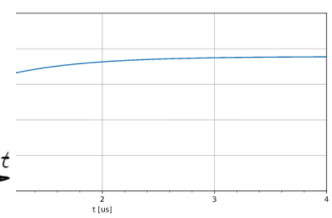
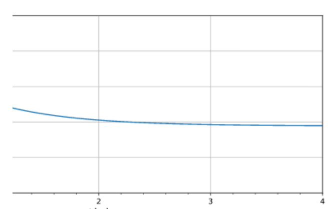


Figure 2



s Time of Exercise Nº 1 on transient Behaviour



Time of Exercise Nº 1 on transient Behaviour

Solution

a) $\underline{\underline{I_{L.0} = 0}}$, $\underline{\underline{U_{o.0} = 0}}$

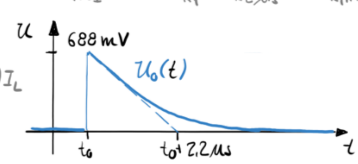
b) $I_{L.\infty} = \hat{U}_s/R_1 = \underline{\underline{1mA}}$; $\underline{\underline{U_{o.\infty} = 0\ V}}$

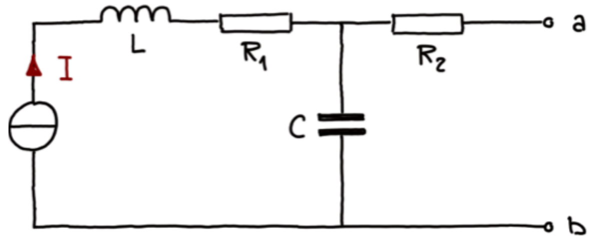
c) $I_L(t) = I_{L.\infty}[1 - e^{-(t-t_0)/\tau}]$, $m_I = U_L(t_0)/L = \frac{R_2}{R_1+R_2}\hat{U}_s/L$, $\tau = \frac{I_{L.\infty} - I_{L.0}}{m_I} = \frac{\cancel{\hat{U}_s}}{R_1} \cdot \frac{(R_1+R_2)L}{R_2 \cancel{\hat{U}_s}} = \frac{L}{R_1 \| R_2} = \underline{\underline{2.2\ \mu s}}$

$(U_s - U_o)/R_1 = I_L + U_o/R_2$ (Kirchhoff's nodal equation)

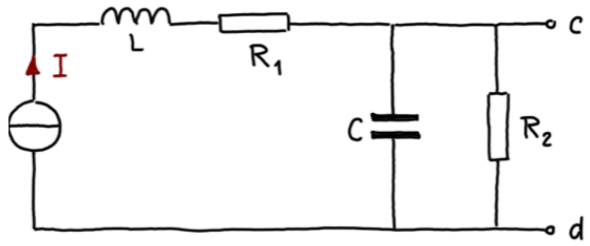
$U_s/R_1 - I_L = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)U_o = \frac{R_1+R_2}{R_1R_2}U_o \Rightarrow U_o = \frac{R_2}{R_1+R_2}U_s - (R_1\|R_2)I_L$

$U_o(t_0^+) = \frac{R_2}{R_1+R_2}\hat{U}_s = \underline{688\ mV}$, $\underline{U_o(t \to \infty) = 0\ mV}$





A



B

a Tension à vide $\underline{U}_{ab}$ et impédance interne $\underline{Z}_{ab}$: $\underline{U}_{ab} = -jI/(2\pi f C) = 2.56\,V\,e^{-j\pi/2}$

$$Z_{ab} = R_2 - j/(2\pi f C) = 330\,\Omega - j\,362\,\Omega$$

b Tension à vide $\underline{U}_{cd}$ et impédance interne $\underline{Z}_{cd}$:

$$\underline{U}_{cd} = \frac{R_2/(j2\pi f C)}{R_2 + 1/(j2\pi f C)} \cdot I = \frac{R_2}{1 + j2\pi f R_2 C} \cdot I = \frac{R_2}{\sqrt{1 + (2\pi f R_2 C)^2}} \cdot I \cdot e^{j \cdot \arctan(-2\pi f R_2 C)} = 1{,}72\,V \cdot e^{-j0.74}$$

$$Z_{cd} = \frac{R_2}{1 + j2\pi f R_2 C} = \frac{R_2}{1 + (2\pi f R_2 C)^2}(1 - j2\pi f R_2 C) = 180\,\Omega - j\,164\,\Omega$$

Consider the circuit shown at figure 7 composed of two capacitors with capacitances C_1 = 1.0 nF and C_2 = 560 pF, of a resistor with resistance R = 330 kΩ and of an ideal voltage-source with complex RMS-voltage $\underline{U}_i$ = 1 V.

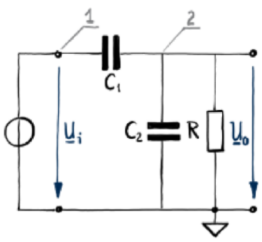


Figure 7: Schematic of Circuit of Exercise Nº 3 on complex Transfer-Functions

1. Establish the equations of the circuit of figure 7 according to the method of nodal-potential-analysis with the potentials V_1 and V_2 of nodes Nº 1 and 2 as unknowns. If you have to introduce additional unknowns, specify and indicated them on the schematic of figure 7.
2. Determine the expression of the transfer-function $\underline{U}_o/\underline{U}_i$, specify and evaluate its characteristic frequencies and represent its level and phase on the corresponding charts hereafter (figures 10 and 11).

The following circuit-equations are obtained by applying the nodal-potential-analysis:

$$\underline{I}_i + j2\pi f C_1(\underline{V}_2 - \underline{V}_1) = 0 \qquad \text{Node 1}$$

$$j2\pi f C_1(\underline{V}_1 - \underline{V}_2) - \underline{V}_2/R - j2\pi f C_2 \underline{V}_2 = 0 \qquad \text{Node 2}$$

$$\underline{V}_1 = \underline{U}_i \qquad \text{Ideal Voltage-Source}$$

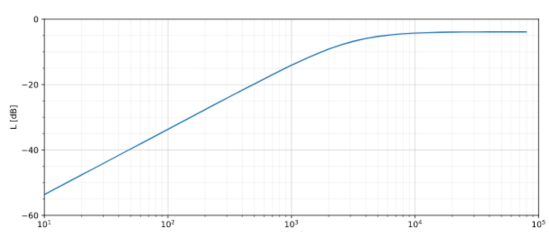
The transfer-function may be derived from the circuit-equations found above and identifying $\underline{V}_2 = \underline{U}_o$ or by using ad-hoc impedance-simplification. The derivation shown hereafter starts from the formula of the voltage-divider and subsequently brings the expression in a more usable form.

$$\underline{A}_V = \frac{\underline{U}_o}{\underline{U}_i} = \frac{R \| [1/(j2\pi f C_2)]}{1/(j2\pi f C_1) + R \| [1/(j2\pi f C_2)]} = \frac{\frac{R/(j2\pi f C_2)}{R + 1/(j2\pi f C_2)}}{1/(j2\pi f C_1) + \frac{R/(j2\pi f C_2)}{R + 1/(j2\pi f C_2)}}$$

$$\underline{A}_V = \frac{R/(1 + j2\pi f RC_2)}{1/(j2\pi f C_1) + R/(1 + j2\pi f RC_2)} = \frac{j2\pi f RC_1/(1 + j2\pi f RC_2)}{1 + j2\pi f RC_1/(1 + j2\pi f RC_2)}$$

$$\underline{A}_V = \frac{j2\pi f RC_1}{1 + j2\pi f R(C_1 + C_2)} = \frac{jf/f_D}{1 + f/f_{P1}}$$

$$\text{with} \qquad f_D = \frac{1}{2\pi RC_1} \qquad f_{P1} = \frac{1}{2\pi R(C_1 + C_2)}$$



Consider the circuit shown at figure 6 composed of three resistors with resistances R_1 = 22 kΩ, R_2 = 10 kΩ and R_3 = 56 kΩ and of a capacitor with capacitance C = 1.0 nF.

1. Evaluate the circuit's resistance $R_{AB.0}$ and reactance $X_{AB.0}$ (the real and imaginary part of its impedance) between the terminals A and B in the case where the frequency f tends towards **zero**.
2. Evaluate the circuit's resistance $R_{AB.\infty}$ and reactance $X_{AB.\infty}$ (the real and imaginary part of its impedance) between the terminals A and B in the case where the frequency f tends towards **infinity**.
3. Evaluate the circuit's resistance $R_{AB}(f_1)$ and its reactance $X_{AB}(f_1)$ at frequency f_1 = 10 kHz.
4. Assuming that the voltage between terminals A and B is a cosine alternating signal with frequency f_1 = 10 kHz and RMS-voltage of U_{AB} = 2 V, determine the active power P dissipated inside the two-terminal-device between terminals A and B.

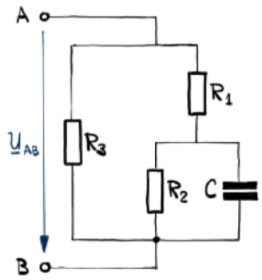


Figure 6: Schematic of Circuit of Exercise Nº 2 on complex Representation of AC-Circuits

1. $R_{AB.0} = R_3 \| (R_1 + R_2) = R_3 \cdot (R_1 + R_2)/(R_1 + R_2 + R_3)$ = 20.4 kΩ $X_{AB.0}$ = 0 Ω
2. $R_{AB.\infty} = R_3 \| R_1 = R_1 R_3/(R_1 + R_3)$ = 38.5 kΩ $X_{AB.\infty}$ = 0 Ω
3. The derivation of the frequency-dependent expression of the impedance $\underline{Z}_{AB}$ is shown below. The following numerical values are obtained for the equivalent resistance R_Π = 20.4 kΩ for the zero-frequency f_z = 23.1 kHz and for the pole-frequency f_p = 18.0 kHz The following numerical values are obtained for th circuit's resistance R_{AB} = 19.3 kΩ for its reactance X_{AB} = -1.94 kΩ and for the modulus of its impedance $|\underline{Z}_{AB}|$ = 19.4 kΩ
4. The derivation of the active power dissipated inside the circuit is shown below. The following numerical value is obtained for said active power P = 205 µW

Starting-point is the series-parallel-connection of impedances that yield the overall impedance:

$$\underline{Z}_{AB} = \frac{R_3 \cdot \underline{Z}_{RC}}{R_3 + \underline{Z}_{RC}} \qquad \text{with} \qquad \underline{Z}_{RC} = R_1 + \frac{R_2/(j2\pi f C)}{R_2 + 1/(j2\pi f C)} = (R_1 + R_2) \cdot \frac{1 + j2\pi f (R_1 \| R_2) C}{1 + j2\pi R_2 C}$$

$$\underline{Z}_{AB} = \frac{R_3 (R_1 + R_2)[1 + j2\pi f (R_1 \| R_2) C]/(1 + j2\pi f R_2 C)}{R_3 + (R_1 + R_2)[1 + j2\pi f (R_1 \| R_2) C]/(1 + j2\pi f R_2 C)}$$

$$\underline{Z}_{AB} = [R_3 \| (R_1 + R_2)] \frac{1 + j2\pi f (R_1 \| R_2) C}{1 + j2\pi f [R_2 \| (R_1 + R_3)] C} = R_\Pi \frac{1 + jf/f_z}{1 + jf/f_p}$$

$$R_\Pi = R_3 \| (R_1 + R_2) \qquad f_z = \frac{1}{2\pi (R_1 \| R_2) C} \qquad f_p = \frac{1}{2\pi [R_2 \| (R_1 + R_3)] C}$$

This result is further developped to find the frequency-dependent expressions of resistance and reactance and of the impedance's modulus:

$$\underline{Z}_{AB} = R_\Pi \frac{(1 + jf/f_z)(1 - jf/f_p)}{1 + (f/f_p)^2} = R_\Pi \frac{1 + f^2/(f_z f_p)}{1 + (f/f_p)^2} + jR_\Pi \frac{f(1/f_z - 1/f_p)}{1 + (f/f_p)^2}$$

$$R_{AB} = R_\Pi \frac{1 + f^2/(f_z f_p)}{1 + (f/f_p)^2}$$

$$X_{AB} = R_\Pi \frac{f(1/f_z - 1/f_p)}{1 + (f/f_p)^2}$$

$$|\underline{Z}_{AB}| = R_\Pi \frac{|1 + jf/f_z|}{|1 + jf/f_p|} = R_\Pi \frac{\sqrt{1 + (f/f_z)^2}}{\sqrt{1 + (f/f_p)^2}}$$

For the active power, the developement is shown hereafter:

$$P = U_{AB} \cdot I_{AB} \cdot \cos\varphi = \frac{U_{AB}^2 \cos[\arg(\underline{Z}_{AB})]}{|\underline{Z}_{AB}|}$$

$$\text{Using} \qquad \cos[\arg(\underline{Z}_{AB})] = \frac{R_{AB}}{|\underline{Z}_{AB}|} \qquad \text{it becomes}$$

$$P = \frac{U_{AB}^2 R_{AB}}{|\underline{Z}_{AB}|}$$