Physique - mécanique

Masse volumique

$$\rho = \frac{m}{V}$$

Dynamique
m, F, E

Statique: $a = 0$, $\sum F = 0$

Cinétique: a, $\sum F = m \cdot a$

Cinématique: x, v, a, t $F, \cancel{E}, \cancel{m}$

Forces

Action : $\sum \vec{F} = m \cdot a$

Réaction : $\vec{F}_{AB} = -\vec{F}_{BA}$

Cinétique

$$\sum \vec{F} = m \cdot \vec{a}$$

$m \cdot a = -F_I$

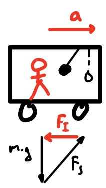
Système accéléré

Observateur dans le syst. accéléré : $\vec{F}_I = -m\vec{a}$

Force d'inertie $\Rightarrow$

Syst. statique

$\sum F + F_I = 0$

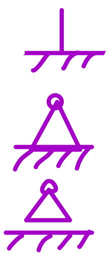


Statique

Appui encastré : $F_\perp$ $F_{//}$ M

Appui fixe : $F_\perp$ $F_{//}$

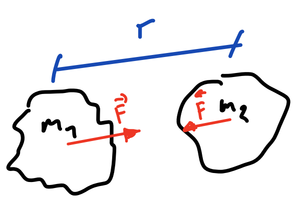
Appui mobile : $F_\perp$



$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_z = 0$$

Gravité

$$\vec{F}_g = G \frac{m_1 \cdot m_2}{r^2}$$

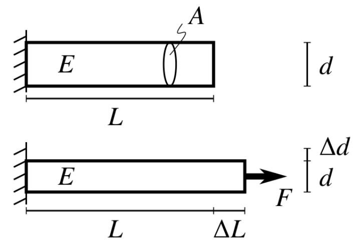


$$F_g = m \cdot g$$

$$g = G \cdot \frac{M_{terre}}{r^2_{terre}} \cong 9{,}81$$

$G = 6{,}67 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}$

Loi de Hooke

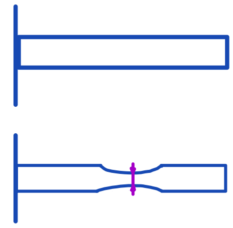


$$\frac{F}{A} = E \frac{\Delta L}{L}$$

Rétrecissement section

$$\frac{\Delta d}{d} = -\nu \frac{\Delta L}{L}$$

↳ coeff. Poisson



Contrainte mécanique

$$\sigma = E \cdot \varepsilon \qquad \varepsilon = \frac{\Delta L}{L}$$

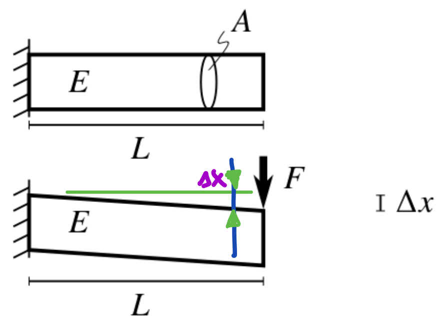
$$\sigma = \frac{F}{A}$$

Cisaillement

$$\tau = G \gamma$$

contrainte cisaill.

$$\gamma \cong \frac{\Delta x}{L} = \tan \gamma$$



$$\boxed{E = 2(1+\nu) G}$$

Relation des coefficients

Ressort

$$F = k \cdot x$$

Ressorts en parallèle : $k = k_1 + k_2$

$$F = F_1 + F_2 = k_1 x + k_2 x$$

Ressorts en série : $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$

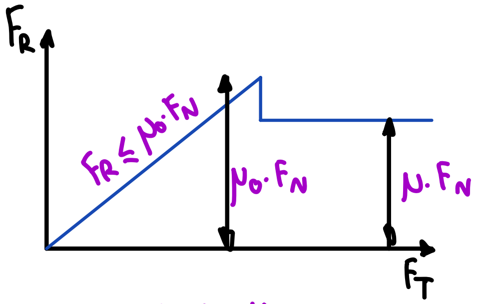
$$\frac{F}{k} = x = x_1 + x_2 = \frac{F}{k_1} + \frac{F}{k_2}$$

Frottement

Statique: $F_R = F_{Traction}$

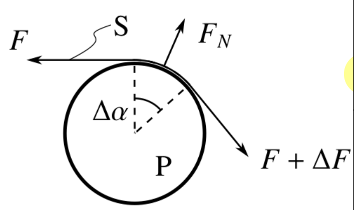
$\Rightarrow \boxed{F_R \leq \mu_0 \cdot F_N}$

Dynamique: $F_R = \mu \cdot F_N$



$\mu_0 > \mu$

Cordes :



La force F_N dépend de l'angle α !

$F_N = 2F \cdot \sin \frac{\Delta\alpha}{2} \Rightarrow \Delta F = \mu F \Delta\alpha$ — petit angle

$$\int_F^{F+\Delta F} \frac{dF}{F} = \int_0^{\Delta\alpha} \mu d\alpha \Rightarrow \ln \frac{F+\Delta F}{F} = \mu \Delta\alpha$$

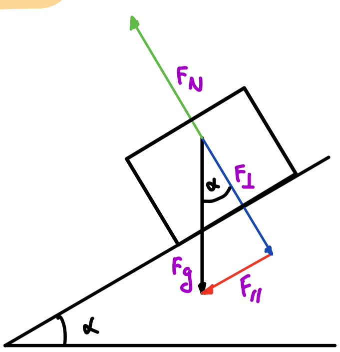
$$\Delta F = F\left(e^{\mu\Delta\alpha} - 1\right)$$

Résistance au roulement

$R_{roul.} \ll F_R$

même formule que frott.
avec coeff. roulement

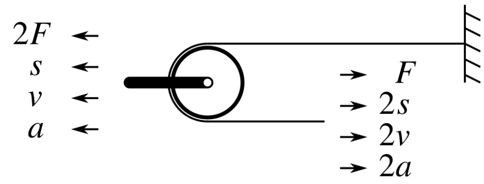
Plan incliné



$F_N = m \cdot g \cdot \cos\alpha$

$F_{//} = m \cdot g \cdot \sin\alpha$

Palans



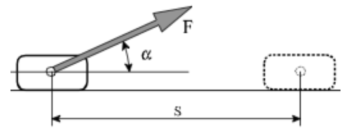
Travail

$W = F \cdot s \cdot \cos\alpha$

$W = \vec{F} \cdot \vec{s}$ {Produit scalaire

$\Rightarrow W = \int_{s_1}^{s_2} \vec{F}(s) \cdot d\vec{s}$ $[W] = J = \frac{kg \cdot m^2}{s^2}$

Travail du frottement:



$W = F_R \cdot s$

Energie potentielle

$E_{pot} = mgh = mg(h_2 - h_1)$

Ressorts : $E = \frac{1}{2} k \cdot x^2$ $x \hat{=}$ distance

Energie cinétique

$E_{cin} = \frac{1}{2} m \cdot v^2$

$$W = \int_{s_1}^{s_2} F\,ds = \int_{s_1}^{s_2} ma\,ds = m\int_{s_1}^{s_2} \frac{dv}{dt}\,ds$$

$$W = m\int_{v_1}^{v_2} \frac{ds}{dt}\,dv = m\int_{v_1}^{v_2} v\,dv = \frac{1}{2}m(v_2^2 - v_1^2)$$

Conservation énergie

$\sum E_1 = \sum E_2$

Pas oublier $E_{frottement}$!

Puissance

$P_m = \frac{\Delta W}{\Delta t}$

$P = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{s}}{dt} = \vec{F} \cdot \vec{v}$

$W = \int P(t)\,dt$

Rendement

$\eta = \frac{E_2}{E_1} = \frac{P_2}{P_1}$

Quantité de mouvement

$\vec{p} = m \cdot \vec{v}$

Changement temporelle:

$$\frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = m \cdot \vec{a} = F$$

Variation de quantité de mouvement = Impulsion

$$\int_{t_1}^{t_2} \vec{F} dt = \vec{p}_2 - \vec{p}_1 = \Delta\vec{p}$$

Conservation

$$\boxed{\sum_{i=1}^{n} \vec{p}_i = \text{const.}} \Rightarrow m_1 V_1 + m_2 V_2 = m_1 U_1 + m_2 U_2$$

⚠ si choc partiellement inélastique, une partie de l'énergie cinétique est perdue !!

mais la conservation s'applique

Choc élastique

La conservation s'applique toujours

$$\underbrace{m_1 V_1}_{\vec{p}_{A_1}} + \underbrace{m_2 V_2}_{\vec{p}_{B_1}} = \underbrace{m_1 U_1}_{\vec{p}_{A_2}} + \underbrace{m_2 U_2}_{\vec{p}_{B_2}}$$

Energie cinétique :

$$\frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 = \frac{1}{2} m_1 U_1^2 + \frac{1}{2} m_2 U_2^2$$

U_1, U_2 vitesse après le choc

$$U_1 = \frac{2 m_2 V_2 + V_1(m_1 - m_2)}{m_1 + m_2}$$

$$U_2 = \frac{2 m_1 V_1 + V_2(m_2 - m_1)}{m_1 + m_2}$$

$$\begin{cases} \text{si } m_1 \ll m_2 \text{ alors } U_1 = -V_1 \\ \text{si } m_1 \gg m_2 \text{ alors } U_2 = 2V_1 \end{cases}$$

Choc inélastique

$U_1 = U_2 = U$ v. identique après choc

$$U = \frac{m_1 V_1 + m_2 V_2}{m_1 + m_2}$$

Perte d'énergie cinétique:

$$E_V = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (V_1 - V_2)^2$$

Choc partiellement élastique : $U_1 \neq U_2$

	$\Sigma \vec{p} = 0$	$\Sigma E = \text{const}$	$U_1 = U_2$
Elastique	✓	✓	✗
Partiel	✓	✗ chaleur...	✗
Inélastique	✓	✗ chaleur...	✓

Cinématique

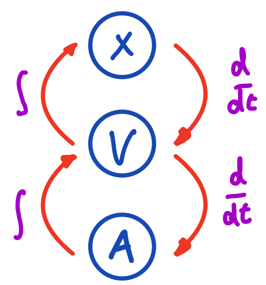
$\vec{V} = \frac{d\vec{r}}{dt}$ $\quad \vec{r}$ position dans l'espace 3D

$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$

Mouvement rectiligne

$V(t) = \int a(t) dt + V_0$

$x(t) = \int v(t) dt + x_0$



Accélération constante:

$V(t) = at + V_0$

$x(t) = \frac{1}{2} at^2 + V_0 t + x_0$

$2a \cdot \Delta s = V_2^2 - V_1^2$

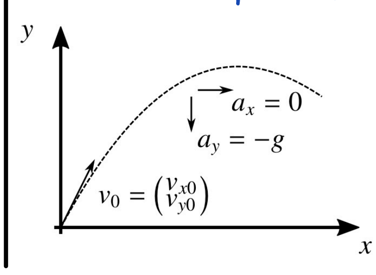
V. moyenne:

$$V_m = \frac{V_1 + V_2}{2}$$

$$a_m = \frac{\Delta V}{\Delta t}$$

Mouvement multi-dim. "balistique"

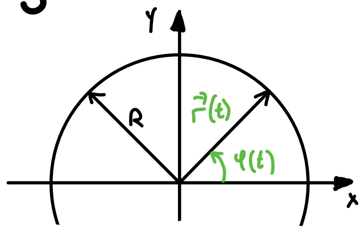
L'objet subit la gravité mais V_x n'est pas modifiée



Mouvement rotatif

$$\varphi(t) = \frac{s(t)}{R}$$

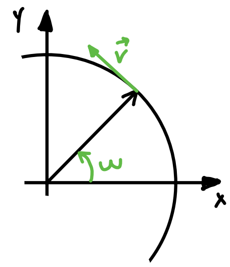
$$\vec r(t) = R\cdot\begin{pmatrix}\cos(\varphi(t))\\ \sin(\varphi(t))\end{pmatrix}$$



Vitesse angulaire

$$\vec v(t) = \frac{d\vec r(t)}{dt} = R\begin{pmatrix} -\frac{d\varphi(t)}{dt}\sin(\varphi(t)) \\ \frac{d\varphi(t)}{dt}\cos(\varphi(t))\end{pmatrix}$$

$$\vec v(t) = \frac{d\vec r(t)}{dt} = R\omega(t)\begin{pmatrix}-\sin(\varphi(t))\\ \cos(\varphi(t))\end{pmatrix}$$



$\Rightarrow v(t) = R\cdot\omega(t)$ valeur absolue

$\omega(t) = \frac{d\varphi(t)}{dt}$ $\quad f = \frac{\omega}{2\pi}$ $\quad T = \frac{1}{f}$ $\quad n = 60f$

$\omega = \frac{2\pi\cdot n}{60}$

Accélération angulaire

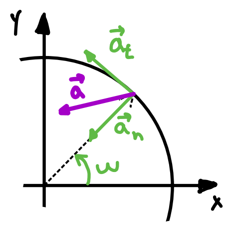
$$\vec a(t) = R\cdot\alpha(t)\begin{pmatrix}-\sin(\varphi(t))\\ \cos(\varphi(t))\end{pmatrix} - R\omega^2(t)\begin{pmatrix}\cos(\varphi(t))\\ \sin(\varphi(t))\end{pmatrix}$$

accélération angulaire: $\alpha(t) = \frac{d\omega(t)}{dt}$

$a_t(t) = R\alpha(t)$

$a_n(t) = R\omega^2(t) = \frac{v(t)^2}{R}$

$a = \sqrt{a_t^2 + a_n^2}$



Cinématique

$\varphi(t) = \frac{1}{2}\alpha t^2 + \omega_0 t + \varphi_0$ t=0

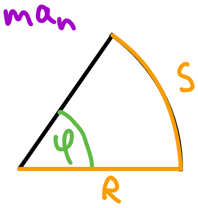
$\omega(t) = \alpha t + \omega_0$

$2\alpha\Delta\varphi = \omega_2^2 - \omega_1^2$

Dynamique

force centripète

$F_N = m a_n$



Relations entre variables

$\varphi = \frac{s}{R}$ $\quad d\vec s = d\vec\varphi \times \vec R$

$\omega = \frac{v}{R}$ $\quad \vec v = \vec\omega \times \vec R$

$\alpha = \frac{a_t}{R}$ $\quad \vec a = \vec\alpha \times \vec R$

$M = R\cdot F$ $\quad \vec M = \vec R \times \vec F$

$L = R\cdot p$ $\quad \vec L = \vec R \times \vec F$

Energie de rotation et inertie

$E_{cin} = \frac{1}{2}mv^2 = \frac{1}{2}mR^2\omega^2$

$J_m = R^2 m$ moment d'inertie d'une masse ponctuelle

$\boxed{E_{rot} = \frac{1}{2}J\omega^2}$ Energie de rotation

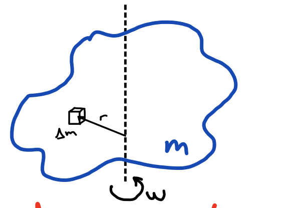
Moment d'inertie

Equivalent à la masse dans un mouvement de translation.

$J = \sum_{i=1}^{N} r_i^2 \Delta m_i$

$J = \int_V r^2\, dm$

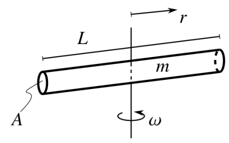
$J = \rho \int_V r^2\, dV$ corps de masse volumique homogène



Moment d'inertie de corps spécifique

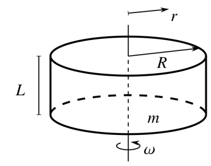
Tige mince:

$J_S = \frac{1}{12}mL^2$



Cylindre:

$J_Z = \frac{1}{2}mR^2$

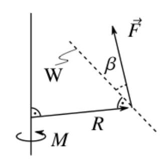


Couple et moment cinétique

Couple: $M = R\cdot F\cdot\cos\beta$ [Nm]

Accélération produit par un couple et un moment d'inertie

↳ $M = J\cdot\alpha$

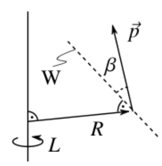


Moment cinétique: $L = R\cdot p\cdot\cos\beta$

(moment de la quantité de mouvement)

$L = J\cdot\omega$

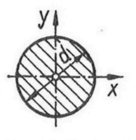
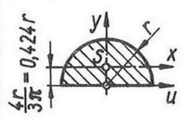
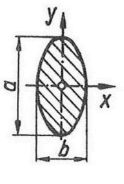
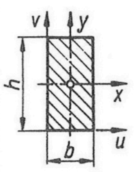
quantité de mouvement

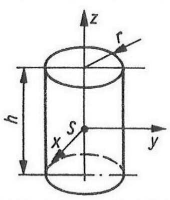


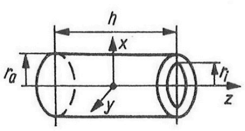
Lois de conservation

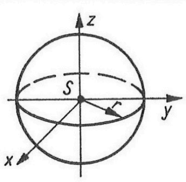
$\sum M = J\cdot\alpha$

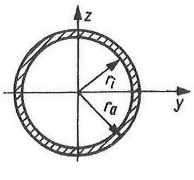
$\sum_{i=1}^{N} L_i = \text{const}$ conservation quantité mouv.

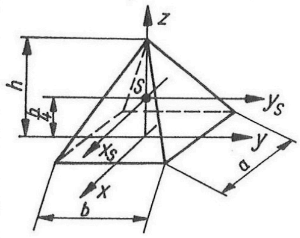
Querschnitt Fläche A	Hauptachsen-Trägheitsmomente I_x, I_y reduziertes Trägheitsmoment I_u	Widerstandsmomente W_x, W_y	Zentrifugalmomente I_{xy}, I_{uy}, I_{uv}
1	2	3	4
	$I_x = I_y = \frac{\pi}{64} d^4$ $= \frac{1}{20} d^4$	$W_x = W_y$ $= \frac{\pi}{32} d^3$ $= 0{,}1 d^3$	$I_{xy} = 0$
$A = \frac{\pi}{4} d^2$			
	$I_x = r^4 \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$ $= 0{,}11 r^4$ $I_y = \frac{\pi}{8} r^4 = 0{,}393 r^4$	$W_x = 0{,}19 r^3$ $W_y = \frac{\pi}{8} r^3$ $= 0{,}393 r^3$	$I_{xy} = 0$ $I_{uy} = 0$
$A = \frac{\pi}{2} r^2$	$I_u = \frac{\pi}{8} r^4 = 0{,}393 r^4$		
	$I_x = \frac{\pi}{64} ba^3$ $= \frac{1}{20} ba^3$ $I_y = \frac{\pi}{64} ab^3$ $= \frac{1}{20} ab^3$	$W_x = 0{,}1 ba^2$ $W_y = 0{,}1 ab^2$	$I_{xy} = 0$
$A = \frac{\pi}{4} ab$			
	$I_x = \frac{1}{12} bh^3$ $I_y = \frac{1}{12} hb^3$	$W_x = \frac{1}{6} bh^2$ $W_y = \frac{1}{6} hb^2$	$I_{xy} = 0$ $I_{uy} = 0$ $I_{uv} = -\frac{1}{4} b^2 h^2$
$A = bh$	$I_u = \frac{1}{3} bh^3$		

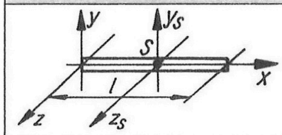
Kreiszylinder	
	$m = \pi \varrho r^2 h$ $J_x = J_y = \frac{1}{4} m \left(r^2 + \frac{h^2}{3}\right)$ $J_z = \frac{1}{2} m r^2$

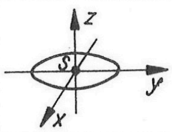
Hohl-Kreiszylinder	
	$m = \pi \varrho h (r_a^2 - r_i^2)$ $J_x = J_y = \frac{1}{4} m (r_a^2 + r_i^2) + \frac{1}{12} m h^2$ $J_z = \frac{1}{2} m (r_a^2 + r_i^2)$ Dünnwandiger Hohlzylinder $r_i \approx r_a \approx r$ $J_x = J_y = \frac{1}{2} m \left(r^2 + \frac{h^2}{6}\right)$ $J_z = m r^2$

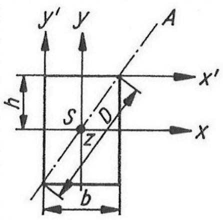
Kugel	
	$m = \frac{4}{3} \pi \varrho r^3$ $J_x = J_y = J_z = \frac{2}{5} m r^2$

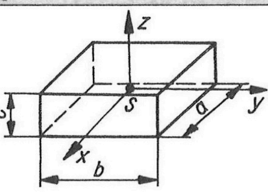
Hohlkugel	
	$m = \frac{4}{3} \pi \varrho (r_a^3 - r_i^3)$ $J_x = J_y = J_z = \frac{2}{5} \frac{m (r_a^5 - r_i^5)}{(r_a^3 - r_i^3)}$ Dünnwandige Hohlkugel $r_i \approx r_a \approx r$ $J_x = J_y = J_z = \frac{2}{3} m r^2$

Gerade Pyramide	
	$m = \frac{1}{3} \varrho abh$ $J_z = \frac{1}{20} m (a^2 + b^2)$ $J_{xS} = \frac{1}{80} m (4b^2 + 3h^2)$ $J_{yS} = \frac{1}{80} m (4a^2 + 3h^2)$

Dünner Stab	
	$J_x = 0$ $J_y = J_z = \frac{1}{3} m l^2$ $J_{yS} = J_{zS} = \frac{1}{12} m l^2$

Mit Masse belegte Kreisscheibe	
	$J_x = J_y = \frac{1}{4} m r^2$ $J_z = \frac{1}{2} m r^2$

Mit Masse belegte Rechteckfläche	
	$J_x = \frac{1}{12} m h^2 \quad J_y = \frac{1}{12} m b^2$ $J_z = \frac{1}{12} m (b^2 + h^2)$ $J_{x'} = \frac{1}{3} m h^2 \quad J_{y'} = \frac{1}{3} m b^2$ $J_A = \frac{1}{6} m \frac{h^2 b^2}{D^2}$

Quader	
	$m = \varrho abc$ $J_x = \frac{1}{12} m (b^2 + c^2)$ $J_y = \frac{1}{12} m (a^2 + c^2)$ $J_z = \frac{1}{12} m (a^2 + b^2)$